A New Analytical Model for Developing Fractional Flow Curve Using Production Data

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Abstract

The immiscible displacement of oil by water through a porous and permeable reservoir rock can be described by the use of a fractional flow curves (f_w versus S_w). Water flooding project parameters can be obtained from the fractional flow curve. However, developing a representative fractional flow curve for a specific reservoir can be quite challenging when fluid and special core analysis data is limited or compromised. Hence, a mathematical model for dependence of f_w on $S_w$is developed by solving material balance algorithm using production data. The results of the model were compared with forecasts from the conventional Bucklett Leverett fractional flow equation and Corey's correlation and were found to be favorable with less time and effort.

Keywords: Waterflooding, Fractional flow curve, Secondary recovery, buckley leverret

Introduction

The displacement of oil by water from a porous and permeable rock is an unsteady-state process because of the change in saturations with time and distance from the injection point (see schematic diagram of Figure 1).These changes in saturation cause the relative permeability values and pressures to change as a function of time at each position in the rock. Figure illustrates the various stages of an oil/water displacement process in a homogeneous linear system.

The mathematical derivation of fluid-flow equations for porous media begins with the simple concept of a material-balance calculation: accumulation equals fluid in minus fluid out. This equation is written for the whole system and for each of the phases: water, oil, and gas. Equations 1 and 2 are the equations for the mass conservation of a water/oil homogeneous linear system:

$$-\frac{\partial}{\partial x}\left(\rho_o u_{ox}\right) = \frac{\partial}{\partial t}\left(\rho_o S_o \phi\right) \tag{1}$$

and

$$-\frac{\partial}{\partial x}\left(\rho_w u_{wx}\right) = \frac{\partial}{\partial t}\left(\rho_w S_w \phi\right), \tag{2}$$

where x is position in x-coordinate system in ft; ρ_o is oil density in lbm/ft^3 or g/cm^3; u_{ox} is oil velocity in the x direction in ft/day; t is time in days; S_o is oil saturation in PV fraction PV; $\varnothing$ is porosity in PV fraction V; ρ_w is water density in lbm/ft^3 or g/cm^3; u_{wx} is water velocity in the x direction in ft/day; and S_w is water saturation jn fraction.

Assuming that the oil and water are incompressible and that the porosity is constant, these equations become:

$$-\frac{\partial q_o}{\partial x} = A\phi\frac{\partial S_o}{\partial t} \tag{3}$$

and

$$-\frac{\partial q_w}{\partial x} = A\phi\frac{\partial S_w}{\partial t}, \tag{4}$$

where q_o is oil-production rate as B/D; A is cross-sectional area available for flowin ft^2; and q_w is water-production rate as B/D. Next, the equations for fractional flow of oil and water are incorporated into these equations. The three fractional-flow equations are:

$$f_o = \frac{q_o}{q_t} = \frac{q_o}{q_w + q_o}, \tag{5}$$

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$$f_w = \frac{q_w}{q_t} = \frac{q_w}{q_w + q_o}, \quad (6)$$

and

$$f_o + f_w = 1.0, \quad (7)$$

where f_o is fractional flow of oil; q_t is the total production rate as B/D; and f_w is fractional flow of water.
Substituting Eq. 6 into Eq. 4 yields:

$$-\frac{\partial f_w}{\partial x} = \frac{\phi A}{q_t}\frac{\partial S_w}{\partial t}. \quad (8)$$

Further mathematical manipulation of these equations obtains the Buckley-Leverett equation (Eq. 9), or frontal-advance equation. To derive this equation, it is assumed that the fractional flow of water is only a function of the water saturation and that there is no mass transfer between the oil and water phases.

$$\left(\frac{dx}{dt}\right)_{S_w} = \frac{q_t}{\phi A}\left(\frac{\partial f_w}{\partial S_w}\right)_t. \quad (9)$$

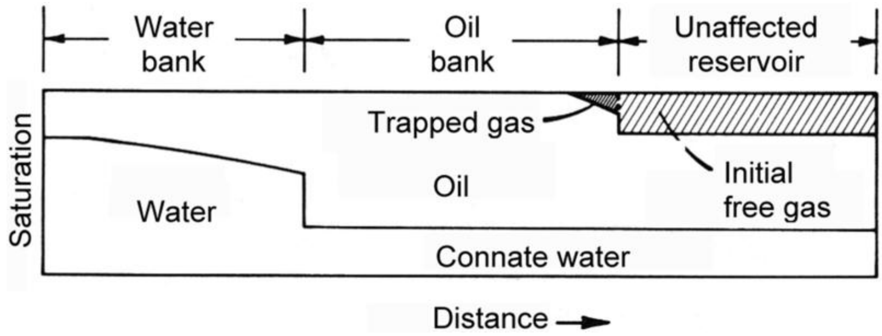

Figure 1: Saturation profile during a water flood. [1]

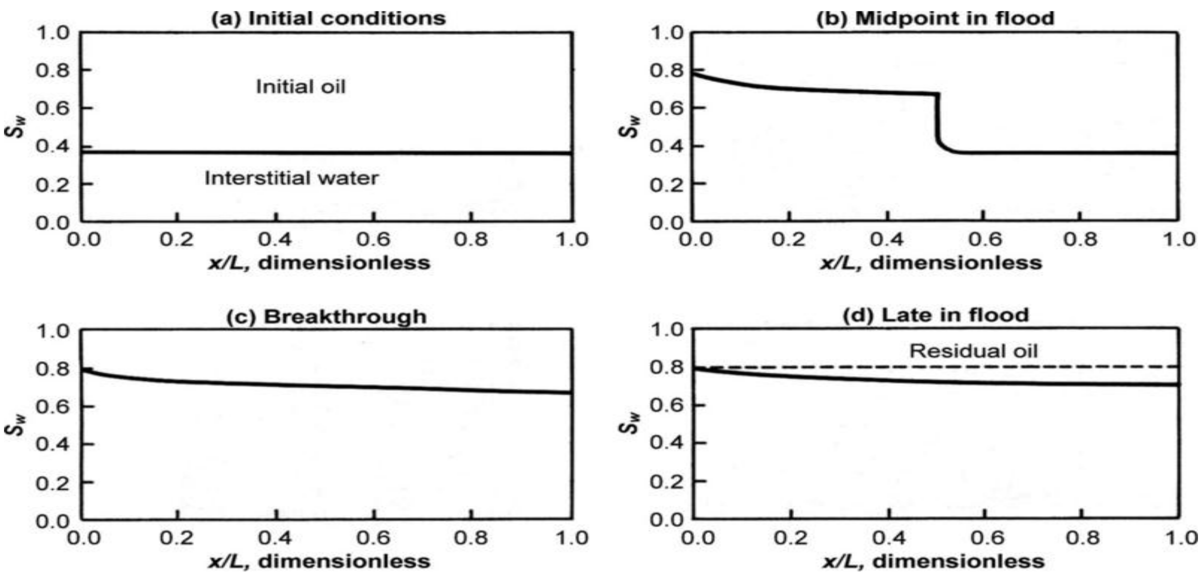

Figure 2: Saturation distribution during different stages of a water flood.[2] **Where L is length (ft); x is x-direction length (ft) and x/L is dimensionless length and varies from 0 to 1.**

This equation shows that in a linear displacement of water displacing oil, each water saturation moves throughout the rock at a velocity which is computed from the derivative of the fractional flow with respect to water saturation.

The general form of the fractional-flow equation for water is:

$$f_w = \frac{1}{1+\left(\frac{k_o}{k_w}\right)\left(\frac{\mu_w}{\mu_o}\right)} + \frac{\frac{k_o A}{\mu_o q_t}\left[\frac{\partial P_c}{\partial x} + (\rho_o - \rho_w) g \sin\alpha\right]}{1+\left(\frac{k_o}{k_w}\right)\left(\frac{\mu_w}{\mu_o}\right)}, \quad (10)$$

where k_o is permeability to oil (darcies); g is gravity constant; α is reservoir dip angle in degrees; and k_w is permeability to water (darcies). This equation includes terms for capillary pressure variation (as a function of saturation) in the linear direction and for the linear system possibly dipping at angle α. Assuming that the gradient in P_c as a function of position is very small and the linear system is horizontal, Eq. 10 reduces to:

$$f_w = \frac{1}{1+\left(\frac{k_o}{k_w}\right)\left(\frac{\mu_w}{\mu_o}\right)} . \quad (11)$$

The curve (f_w versus S_w) derived from fractional flow theory can be used to describe the mechanisms of immiscible [3-9] and miscible flows [10-12]. Developing a representative fractional flow curve for a specific reservoir can be quite challenging when fluid and special core analysis data is limited. Therefore this study is designed to develop a robust mathematical model from production data.

Model development

Correlations for predicting water cut in oil reservoirs could be divided into three main classes: (1) using fractional flow theory, in which relative permeability functions are approximated to establish water cut (or water-oil ratio) variation with oil recovery; (2) using the Arps model and its modifications, for example, semi-log water cut versus oil recovery; and (3) observed trends, for example, linear water cut versus oil recovery [3]. While these methods have been applied extensively, few have been found to be sufficiently robust. Moreover, only the relationship between water cut and cumulative oil production is established in the traditional water cut models. Unfortunately, cumulative oil production itself must be estimated. Considering the aforementioned problems, we derived new models that directly correlate water cut and production time. The production data from a low permeability oil field were used to test the new models.

Fractional flow equation is a qualitative model to determine fraction of total fluid flow for a certain time and in a place with linear water injection system. It describes the relationship of the total flow water in any point of a reservoir at assumed water saturation [3]. The major assumptions are:

- A one dimensional homogenous system
- An isothermal porous medium
- Two phase flow

In the Sitorus model, the Corey equation was applied to the fractional flow equation and assuming every oil withdrawal in time t was replaced by equivalent water from aquifer. Moreover a relationship between cumulative oil production and water cut of wells was developed by matching the calculated to measured water cut mathematically. This approach results in many plausible solutions requiring a lot of caution. However, in the present model, the fractional flow equation was developed from the material balance equation and the relationship between the change in pressure and rock-fluid properties was established.

The relationship between the volumetric flow rate and quantity in volume can expressed as

$$V = \frac{q}{t} \quad (12)$$

Where

V = volume

q = flow rate

t = time

Therefore fractional flow of water in an immiscible flow of oil and water system can written in term of volumetric change as function of pressure

$$f_w = \frac{V_w\,dp}{V_w dp + V_o dp} \tag{13}$$

From material balance equation for oil, it can be say that:

Oil present initially in the reservoir – Oil produced = Oil remaining in the reservoir finally

Or

$$N - N_p = \frac{V_p\,S_o}{B_o} \tag{14}$$

$$N_p = N - \frac{V_p\,S_o}{B_o} \tag{15}$$

The total initial volume of hydrocarbon of the system is then given by:

Initial oil volume + initial gas cap volume = $(PV)(1 - S_{wi})$

$$N_p = N_i - N_r \tag{16}$$

Since they are all functions of pressure, we will have:

$$\frac{d}{dp}N_p = \frac{d}{dp}N_i - \frac{d}{dp}\frac{S_o V_p}{\beta_o} \tag{17}$$

Where N_i = OIIP (oil initial in place).

From the derivation of Muskat equation, let V_p be reservoir pore volume in barrels. Then, the stock tank barrels of oil remaining (N_r) at any pressure is given by [13]:

$$N_r = \frac{S_o V_p}{\beta_o} \text{stock tank barrels [2]} \tag{18}$$

Differentiating N_r in equation 6 with respect to pressure, results in:

$$\frac{dN_r}{dp} = V_p\left(\frac{1}{\beta_o}\frac{d}{dp}S_o - \frac{S_o}{\beta_o^{\,2}}\frac{d}{dp}\beta_o\right) \tag{19}$$

Combining equation 17 and 19 gives:

$$\frac{d}{dp}N_p = \frac{(V_p)(1-S_{wi})}{\beta_{oi}} - V_p\left(\frac{1}{\beta_o}\frac{d}{dp}S_o - \frac{S_o}{\beta_o^{\,2}}\frac{d}{dp}\beta_o\right) \tag{20}$$

For water produced, Net water influx is equal to $W_e - W_p\beta_w$ (21)

$$W_p\beta_w = W_e - W_r \tag{22}$$

From the Pot Aquifer Model, we have:

$$W_e = (C_w + C_f)W_i f(p_i - p) \tag{23}$$

Let $\Delta p = p_i - p$, therefore:

$$W_p = \frac{W_e}{\beta_w} - \frac{S_w V_p}{\beta_w} = \frac{(C_w + C_T)W_i f \Delta p}{\beta_w} - \frac{S_w V_p}{\beta_w} \tag{24}$$

Calculating the initial volume of water in the aquifer requires the knowledge of aquifer dimension and properties. These, however, are seldom measured since wells are not deliberately drilled into the aquifer to obtain such information. For instance, if the aquifer shape is radial, then:

$$W_i = \frac{\pi(r_a^{\,2} - r_e^{\,2})h\emptyset}{5.615} \tag{25}$$

Combining Equations 24 and 25, we have:

$$W_p = \frac{(C_w + C_T)\pi(r_a^{\,2} - r_e^{\,2})h\emptyset f\Delta p}{5.615\beta_w} - \frac{S_w V_p}{\beta_w} \tag{26}$$

$$f = \frac{\emptyset}{360^0} \tag{27}$$

let

$$K = W_e = \frac{(C_w + C_T)\pi(r_a^{\,2} - r_e^{\,2})h\emptyset f\Delta p}{5.615\beta_w} \tag{28}$$

Since they are all function of pressure, it can be concluded that:

$$\frac{d}{d_p}W_p = Kd_p - \frac{d}{d_p}\frac{S_w V_p}{\beta_w} \tag{29}$$

$$\frac{d}{d_p}\frac{S_w V_p}{\beta_w} = V_p\left[\frac{1}{\beta_w}\frac{d}{d_p}S_w - \frac{S_w}{\beta_w^{\,2}}\frac{d}{d_p}\beta_w\right] \tag{30}$$

$$\frac{d}{d_p}W_p = Kd_p - V_p\left[\frac{1}{\beta_w}\frac{dS_w}{d_p} - \frac{S_w}{\beta_w^{\,2}}\frac{d\beta_w}{d_p}\right] \tag{31}$$

From equation 6, since W_p and N_p are functions of pressure, we will have:

$$f_w = \frac{\frac{d}{dp}W_p}{\frac{d}{dp}W_p + \frac{d}{dp}N_p} \tag{32}$$

Putting equation 20 and 31 into 32, results:

$$f_w = \frac{Kd_p - V_p\left[\frac{1}{\beta_w}\frac{dS_w}{dp} - \frac{S_w}{\beta_w^{\,2}}\frac{d\beta_w}{dp}\right]}{Kd_p - V_p\left[\frac{1}{\beta_w}\frac{dS_w}{dp} - \frac{S_w}{\beta_w^{\,2}}\frac{d\beta_w}{dp}\right] + \frac{(V_p)(1-S_{wi})}{\beta_{oi}} - V_p\left(\frac{1}{\beta_o}\frac{dS_o}{dp} - \frac{S_o}{\beta_o^{\,2}}\frac{d\beta_o}{dp}\right)} \tag{33}$$

Truncating common terms gives f_w as:

$$f_w = \frac{Kd_p - \frac{V_p}{\beta_w}\frac{dS_w}{d_p}\left[1 - \frac{S_w}{\beta_w} * \frac{d\beta_w}{d_p}\right]}{Kd_p - \frac{V_p}{\beta_w} * \frac{dS_w}{d_p}\left[1 - \frac{S_w}{\beta_w} * \frac{d\beta_w}{d_p}\right] + \frac{(V_p)(1-S_{wi})}{\beta_{oi}} - \frac{V_p dS_o}{\beta_o\,dp}\left(1 - \frac{S_o}{\beta_o}\frac{d\beta_o}{dS_o}\right)} \tag{34}$$

$$\frac{(V_p)(1 - S_{wi})}{\beta_{oi}} = \text{OIIP}$$

$$f_w = \frac{KdP - \frac{V_p}{B_w}\frac{dS_w}{dP}\left[1 - \frac{S_w}{B_w} * \frac{dB_w}{dP}\right]}{KdP - \frac{V_p}{B_w}\frac{dS_w}{dP}\left[1 - \frac{S_w}{B_w} * \frac{dB_w}{dP}\right] + OIIP - \frac{V_p dS_o}{B_o\,dP}\left(1 - \frac{S_o}{B_o}\frac{dB_o}{dS_o}\right)} \tag{35}$$

Table 1: FX Reservoir Characteristics

Property	Value
Discovered/Streamed	1965/1968
Inj. Start Date	Aug 1991
Datum, ft subsea	-5300
Average thickness, ft	87
Average porosity, %	31.2
Average Permeability, mD	1210
Swi, avg, %	**34**
Sorw, %	**25**
Pi, psig	2203
Pb, psig	2171
Oil fvf, rb/stb	1.200
Rsi, scf/stb	336
Oil visc, cp	1.47
Oil grav, API	27.3
OOIP, MMSTB	252
Cum Inj, MMBWI	127
Current R_f, %	**42**
Ultimate R_f, %	**51**
Gas sat. at start of inj. (assuming no segregation*), %	17
m ratio (G/N)	0.05

Source: SEDECO OIL COMPANY

Results and discussions

It is very important to establish how the change in pressure in a reservoir is affected by varying reservoir parameters/ rock-fluid properties. These relationships can be defined from the data and plots presented below. Table 1 shows the input data/ reservoir characteristics that were used to develop the present model. However, certain parameters such as oil and water formation volume factors are not shown and should not be considered as input data.

Figure 1 shows the relationship between saturation of water and pressure. Figure 1 is characteristic of a logarithmic function and represents the best description about the dependence of S_w on pressure. For small values of pressure S_w are negative and for large pressures they are positive but stay small. Tangents of the ratio were taken at different points to determine $\frac{dS_w}{d_p}$. $\frac{dS_w}{d_p}$ is high at low pressures while it is low at high pressures. From the plot that as pressure declines, it can be seen that the saturation of water declines. However the decline is sharper at low range of pressure. This means that the faster the energy of the reservoir is depleted, the more oil is expelled from the pores of the reservoir.

Figure 2 shows a plot of water formation volume factor (FVF) as a function of pressure in the reservoir. As the pressureis reduced below the initial reservoir pressure (p_i), the oil volume increasesdue to the oil expansion. This behavior results in an increase in the oil formationvolume factor and will continue until the bubble-point pressure isreached. At P_b, the oil reaches its maximum expansion and consequentlyattains a maximum value of

B_{ob} for the oil formation volume factor. As thepressure is reduced below P_b, volume of the oil and Bo are expected to decrease as thesolution gas is liberated, but the FVF still increases because the shrinkage of the water resulting from gas liberation is insufficient to counterbalance the expansion of the liquid. This is the effect of the small solubility of natural gas in water. Since the plot however gives a straight-line curve, $\frac{d\beta_w}{d_p}$ would be constant at all points as pressure changes.

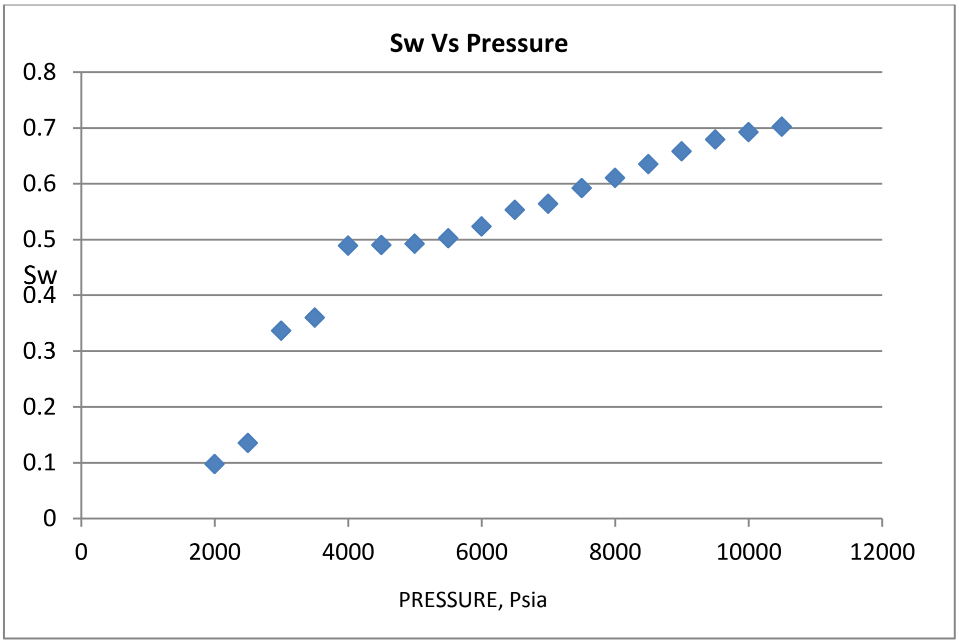

Figure 1: The changes of water saturation (Sw) against pressure

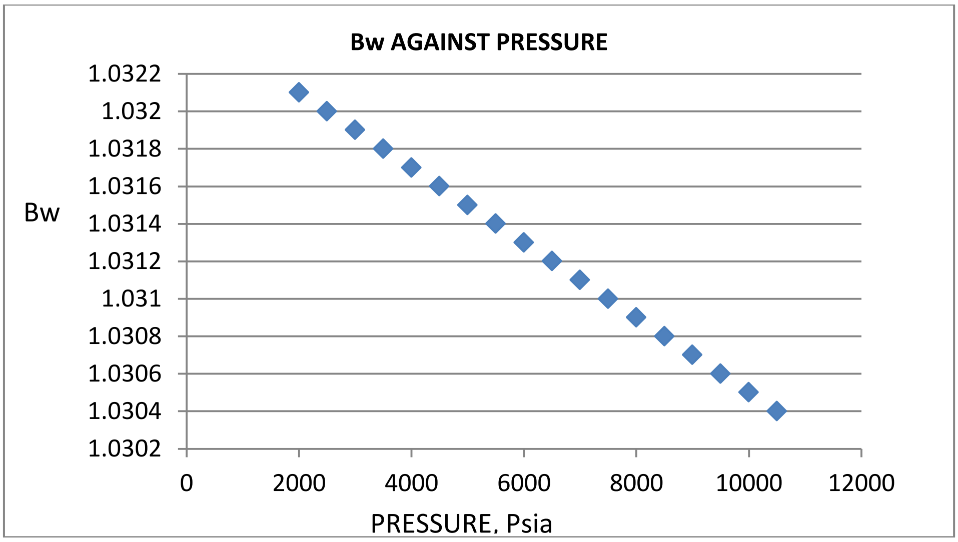

Figure 2: The changes of water FVF against pressure

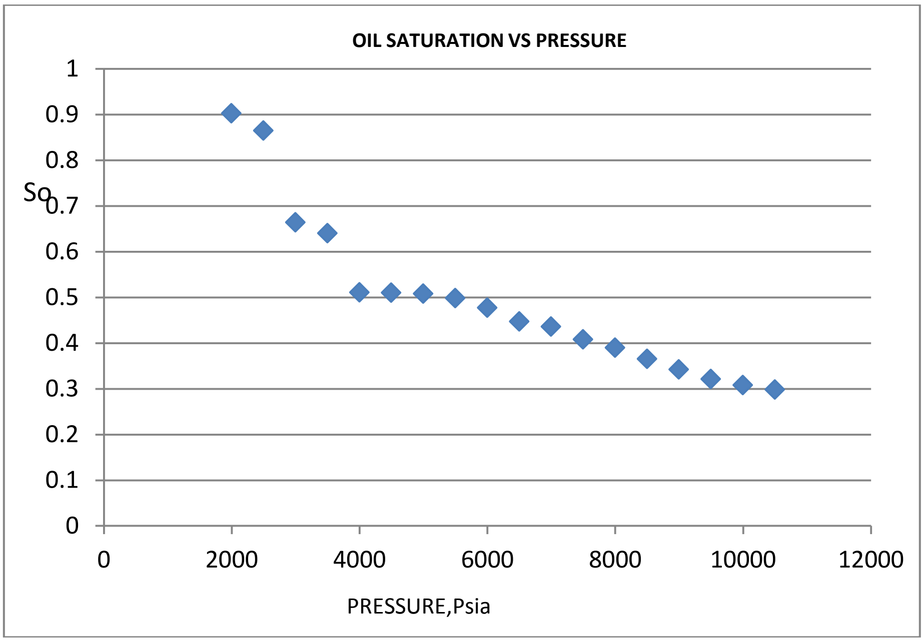

Figure 3: The changes of oil saturation (So) against pressure

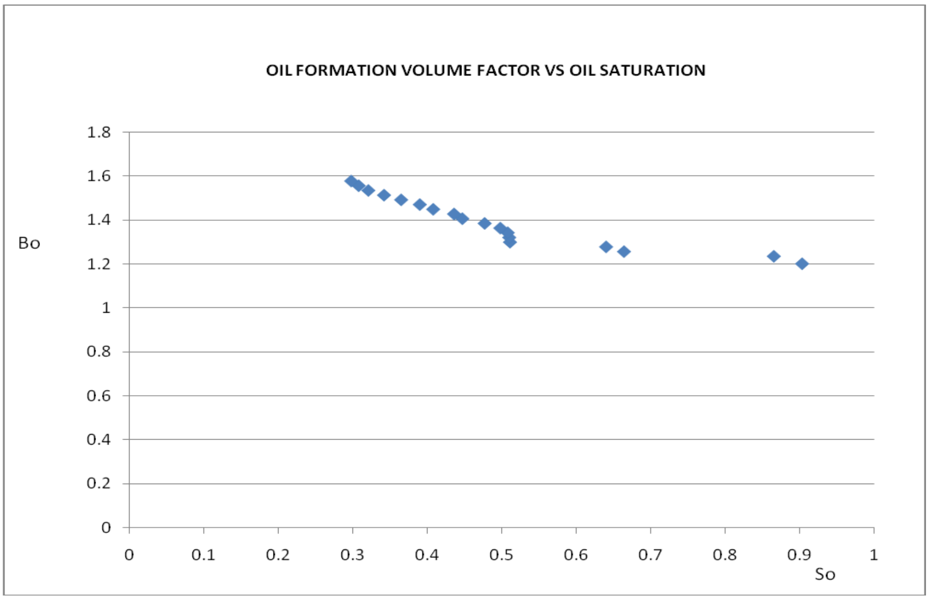

Figure 4: The changes of oil FVF against oil saturation

Figure 3 shows the relationship between oil saturation (S_0) and pressure. As pressure decreased, S_0 increased linearly exhibiting two slopes which the smaller slope observed at high range of pressure. There exists a distinct break at about 4000psia delineating high liquid expansion below and slight liquid expansion above the break.

Figure 4 shows the relationship between oil formation volume factor and saturation. As the oil saturation decreased due to the shrinkage of oil below P_b, B_o also slightly increased though not linearly. Since it is not a straight line graph, the values of $\frac{dB_o}{dS_o}$ would vary with pressure.

Comparison of fractional flow curve

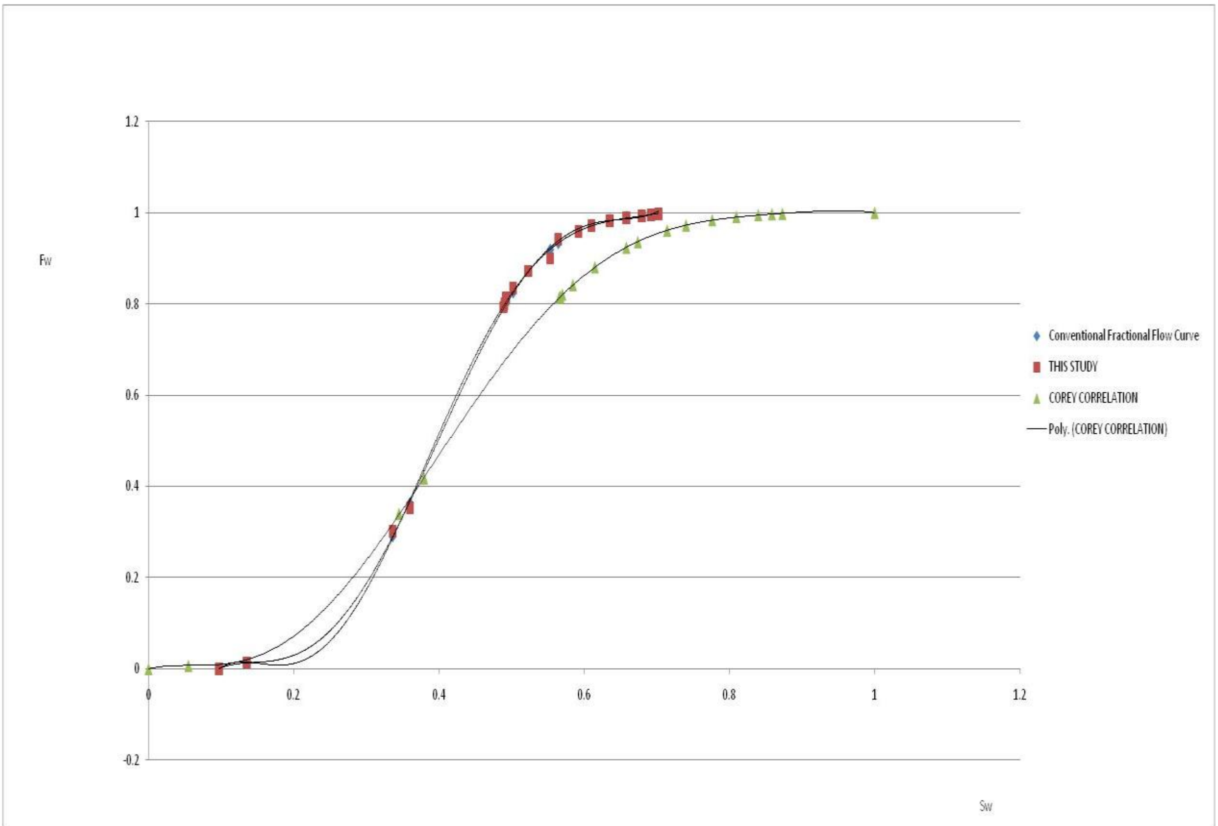

Figure 5: Comparison of Conventional and Present Models

Figure 5 shows the various fractional flow curves developed from different studies. The initial point is the connate water saturation. The connate water saturation is primarily important because it reduces the amount of space available for oil and gas. It is generally not uniformly distributed throughout the reservoir but varies with permeability, lithology, and height above free water table.

The curve developed from this study, shows the solutions for the fractional flow curve in terms of water saturation and pressure profile extracted from the production data given in Table 2. It is also noticed that the profile predicted by the fractional flow curve developed from the present model is more accurate than the Corey's correlation in contrast with the conventional fractional flow curve.

Conclusion

In this paper, the proposed model was used to develop the fractional flow curve for a water flooded reservoir using production data. Fractional flow curve was analysed in terms of water saturation and pressure profile. The results were compared with the conventional fractional flow equation of Buckley Leverett and Corey's correlation. There is good agreement between the conventional method and the curve developed from this study. With the model developed in this study, the challenge of developing a representative fractional flow curve for a specific reservoir can easily be overcome especially when fluid and special core analysis data is limited or compromised or in cases where core samples cannot be easily (deep offshore) retrieved. This method also helps to save time.

Table 2: Comparison of the methods

COREYS METHOD		THIS STUDY		CONVENTIONAL METHOD	
Water Saturation, Fraction	fw	Water Saturation, fraction	fw	Water Saturation, fraction	fw
0	0.00000	0.097	-0.0000014	0.097	0.00000
0.0548	0.007305	0.135	0.0140519	0.135	0.014054209
0.3449	0.339897	0.336	0.30099784	0.336	0.291026807
0.3795	0.418179	0.360	0.3534084	0.36	0.363437438
0.5657	0.813962	0.489	0.79298213	0.489	0.792441408
0.5671	0.816205	0.490	0.79998226	0.49	0.79646246
0.5700	0.82064	0.492	0.81348307	0.492	0.800439034
0.5844	0.841769	0.502	0.83408482	0.502	0.825842697
0.6147	0.880551	0.523	0.87198825	0.523	0.871774323
0.6580	0.923571	0.553	0.89999102	0.553	0.9190879
0.6739	0.936042	0.564	0.9412946	0.564	0.933642999
0.7143	0.960945	0.592	0.95949647	0.592	0.961033901
0.7403	0.972531	0.610	0.97099757	0.61	0.973509934
0.7763	0.984072	0.635	0.98299871	0.635	0.984924623
0.8095	0.991042	0.658	0.98899928	0.658	0.991589571
0.8398	0.995134	0.679	0.99349965	0.679	0.995379591
0.8586	0.996843	0.692	0.9958998	0.692	0.99695873
0.8730	0.99782	0.702	0.99759989	0.702	0.997796192
1	1				

Recommendation

- The accuracy of model depends on PVT and production data
- With this method, the ability to determine saturation of water at corresponding pressure decline
- This model can be applied in critical areas or harsh regions (deep water offshore) where it is difficult to obtain core samples.
- This model can also be applied where there is a case of poor core handling.

Nomenclature

x = position in x-coordinate system, ft;
α = dip angle
f_{ws} = surface water cut, STB/STB
W_e = cumulative water influx, bbl
W_p = cumulative water produced, stb
$\sin\alpha$ = positive for updip flow and negative for down dip flow
$\Delta\rho$ = water-oil density difference
i_w = water injection rate
k_{ro}, k_{rw} = relative permeability
WOR_s = surface water-oil ratio, STB/STB
WOR_r = reservoir water-oil ratio, STB/STB
ρ_o,ρ_w = density, lbm/ft^3 or g/cm^3;
u_{ox}, u_{wx} = velocity in the x direction, ft/day;
t = time, days;
S_o, S_w = saturation, fraction PV;
$\emptyset$ = porosity, fraction BV;
f_o, f_w = fractional flow;
q_t = the total production rate, B/D;
q_o, q_w = production rate, B/D;
A = cross-sectional area available for flow, ft^2;
k_o, k_w = effective permeability, darcies;

g = gravity constant;
α = reservoir dip angle, degrees;
k_w = permeability to water, darcies.
μ_o, μ_w = viscosity
B_w, B_o = formation volume factor, bbl/STB
P_o, P_w = Pressure, psi
μ_w, μ_o = viscosity, cP
V = volume, ft^3
c_w = water compressibility, 1/psi
c_f = formation compressibility, 1/psi
c_t = total compressibility, 1/psi
r_a = apparent wellbore radius, ft
r_e = reservoir radius, ft
h = reservoir thickness, ft
N = stock tank oil initially in place (stb)
N_p =cumulative oil recovery (stb)
PV = pore volume injected
N_i = OIIP (oil initial in place)

Subscripts

α = phase label
g = gas phase
o = oil phase
w = water phase

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